GRADUATE COURSE

Fall 2018

ME710: Higher Order Numerical Methods for PDEs (3-0) 3

Prerequisite : Instructor's Consent.

This course is about one of the big three technologies for the numerical solution of DEs as follows:

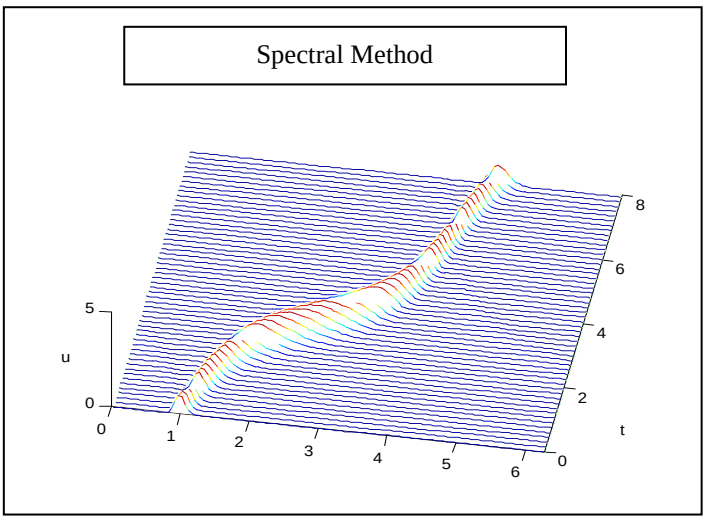
1950s: Finite Difference Methods
1960s: Finite Element Methods
1970s: Spectral Methods.

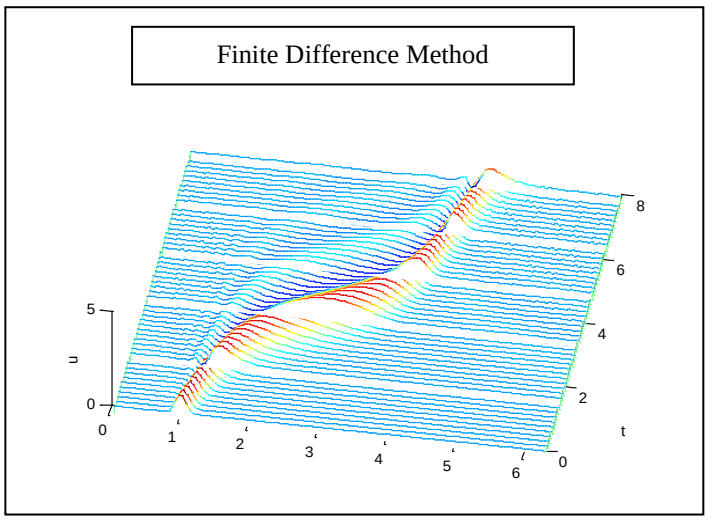
If one wants to solve an ODE or PDE to high accuracy, and if the data defining the problem are smooth, then spectral methods are usually the best tool.

For example, consider the variable coefficient wave equation

$$u_t + c(x)u_x = 0, \quad \text{for} \quad x \in [0, 2\pi], \quad t > 0 \quad \text{where} \quad c(x) = 1/5 + \sin^2(x-1)$$

with periodic boundary conditions and initial condition $u(x,0) = \exp(-100(x-1)^2)$. In the following figures, we compare the numerical solutions obtained by applying two methods at the same spatial and temporal resolution. The wave remains coherent and clear with no ripples in the case of spectral method indicating the high accuracy obtained.





This course will present some of the fundamental ideas and techniques of spectral methods. The emphasis is on spectral collocation ("pseudospectral") methods. The teaching teachnique will be based on problem solving. There will be many examples and case studies drawn from various fields.

MATLAB will be the main computational tool throughout the course. With this tool, one can present advanced numerical algorithms and solutions of nontrivial problems in complete detail with great brevity. This course will be a good opportunity to get acquainted with and get fluent in MATLAB.

Topics List

- Fourier - Spectral Differentiation
- Chebyshev - Spectral Differentiation
- Boundary Value Problems
- Initial Value Problems
- Polar Coordinates
- Iteration and Preconditioning
- Spectral Element Method
- Discontinuous-Galerkin Method

Some Refs. :

- *Spectral methods in MATLAB* by Trefethen, 2000.
- *Spectral/hp Element Methods for Computational Fluid Dynamics* by Karniadakis & Sherwin, 2005.
- *Spectral Methods : Fundamentals in Single Domains* by Canuto & Hussaini & Quarteroni & Zang , 2006.
- *Spectral Methods for Time-Dependent Problems* by Hesthaven & Gottlieb, 2007.
- *Nodal Discontinuous Galerkin Methods* by Hesthaven & Warburton, 2008.

Course Website : *http://users.metu.edu.tr/tarman/ME710/*